and Norman the inclusion of the new operator(s) in the set $L$ and the new $g$ functions accounts for the interaction between the original $H_1$, $H_2$, and $H_3$, (and other $H_k$ which may exist in the original $H$ operator).

The 15-degree migration operator can be written in a form that contains three operators $H_1$, $H_2$, and $H_3$. A reasonable modification to one of the $x$-dependent terms of the equation produces a set $L$ with a finite number of elements. The resulting set $L$ contains an additional operator $H_4$. It turns out that $H_1$ corresponds to the usual downward continuation operator and $H_2$ corresponds to the usual time shift operator familiar from the split-step approximation. $H_3$ is present only in a formulation which allows lateral velocity variations to appear explicitly in the differential operator. $H_3$ has a negligible effect on the solution in the neighborhood of the finite difference operator. However, the interaction of $H_3$ with $H_2$ and $H_1$ is responsible for the existence of the new operator $H_4$ which generates a lateral shift of the field proportional to both the depth step $\Delta z$ and to the velocity gradient. The details of the solution are too involved to present here but will shortly be submitted for publication.

An advantage of the operator formulation is that it is clear that operators which commute do not interact. For example, in the 15-degree migration operator the operator $H_2$, corresponding to the time shift correction for laterally varying velocity, commutes with all of the other operators. In particular it commutes with $H_1$ the downward continuation operator. Thus, the time shift may be applied either before or after the field is downward continued.

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## Three Common-Shot Migration Methods S5.2

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It has long been recognized that conventional CDP processing degrades in geologic areas with steeply dipping beds and strong lateral velocity variation. As a result common-shot processing has been suggested as an alternative in such regions. A requisite for successful processing of this type is a common-shot migration method which can be used with arbitrary velocity variation. In this paper we describe three common-shot migration methods which should perform well in such complicated areas. The first migration algorithm is a ray tracing method which maps digitized horizons from $x$-$t$ space $x$-$z$ space. The other two methods operate on common-shot gathers and map time sections into depth sections. The common departure point for the three methods is an imaging principle which replaces the exploding reflector model used in the migration of stacked sections.

We describe the three migration methods and present synthetic examples which shed light on their features.

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### Ray tracing migration

Common-shot ray tracing migration is very similar to ray tracing migration of stacked time sections. Selected digitized horizons are downward continued into the earth along rays. The angle of emergence of the rays at each geophone is determined from the slope of the traveltime curve under the assumption that the interval velocities between the reflection events on the time section are known. In this study we assumed constant interval velocities between the layers in the subsurface, although the method can be generalized to account also for velocity gradients. The point of departure from migration of stacked sections is that the length of the extrapolated ray no longer corresponds to the total traveltime from the reflector to the surface, but rather the condition becomes that the sum of the traveltimes along the ray from the shot to the reflector and along a second ray from the reflector to the geophone must equal the measured traveltime of the event in the time section (Figure 1). The determination of the point ($F$) for which the condition is met involves a search procedure among different rays from the shot location (e.g., $SF_1$, $SF_2$ in Figure 1).

### Full acoustic migration

In full acoustic common-shot migration, the acoustic wave equation is downward continued in depth. Based on the statement that a reflector exists whenever the direct wave from the shot and the reflected wave are time coincident, the final depth section is obtained by maintaining the wave amplitudes at the time of arrival of the direct wave. Let $P(x, y = 0, t)$ denote the recorded common-shot gather on the Earth's surface. The final migrated section then consists of the field $P[x, y, t_d(x, y)]$, where $t_d(x, y)$ denotes the time of arrival of the direct wave from the shot to the point $(x, y)$ in the subsurface. In space frequency migration methods, $\bar{P}(x, y, \omega)$ is calculated for each frequency based on the surface values $\bar{P}(x, y = 0, \omega)$ and the final depth section is then given by

$$\bar{P}(x, z, t_d) = \sum_{w} \bar{P}(x, y, \omega)\, e^{i\omega t}d, \qquad (1)$$

where the summation is carried out over the seismic frequency band. Except for the different imaging condition (1), common-shot depth migration is identical to the migration of zero offset or stacked sections. The input for the migration consists of the common-shot gather $P(x, y = 0, t)$, the

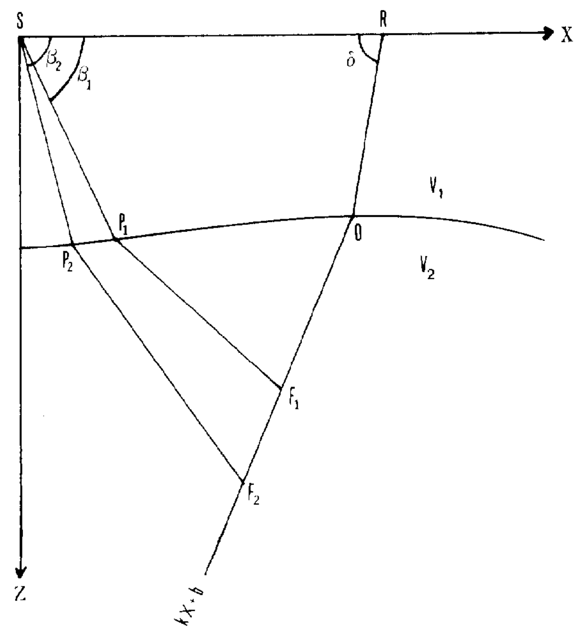


FIG. 1.

acoustic velocity $C(x, z)$ and the "imaging time" $t_d(x, y)$. The output yields the depth section $P(x, y, t_d)$.

The success of common-shot migration depends upon an efficient method for calculating $t_d(x, y)$. In principle $t_d$ can be calculated by ray tracing from the shotpoint to each point $(x, y)$ in the subsurface (or more precisely to the discrete set of points $x_i y_i$ of the numerical mesh). However, this ray tracing can become time consuming and we chose instead to calculate $t_d$ by a direct solution of the eikonal equation.

### The eikonal equation

In a two-dimensional acoustic medium with variable velocity $C(x, y)$, the eikonal equation is given by

$$\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2 = \frac{1}{C^2}$$

where $T$ denotes the traveltime to a given point in the subsurface. For the numerical integration, the equation is rewritten explicitly as

$$\frac{\partial T}{\partial y} = \frac{1}{C^2} - \left(\frac{\partial T}{\partial x}\right)^2$$

The numerical solution proceeds as an integration in depth after a specification of $T(x, t = 0)$ on the surface. This specification can be calculated directly in the case of uniform surface velocity or otherwise by ray tracing. In the numerical integration the term $\partial T/\partial x$ is calculated by second-order differencing. During the integration, careful attention must be given to the possibility of existence of discontinuous wavefronts as in the case of postcritical incidence angles on an interface.

### Two-eikonal method

Through a solution of the eikonal equation, the traveltime from a point on the surface to all points in the subsurface can be obtained. The migration consists of constructing solutions of the eikonal equation from the shot location and from each geophone location. The amplitude of each sample of the depth section is calculated by adding contributions from all the geophone traces. For a given geophone trace and depth point, the amplitude contribution is equal to the amplitude of the trace at a time equal to the sum of the times from the solutions of the eikonal equation for the shot location and for the geophone location, respectively.

Although the two-eikonal method does not reproduce amplitude values as accurately as the wave equation method, it is fast and easy to implement. Because the method does not require spatial numerical differentiation, it can also work with irregular geophone spacing. This point may have added significance in implementing the method in three dimensions.

The two-eikonal method is easiest to visualize in a constant velocity medium. The contours of the solutions of the eikonal equation then give families of semicircles centered at the corresponding geophone or shot location. A time section consisting of a single spike at time $t_1$ at one of the geophones will map into an ellipse with focii at the shot and geophone locations respectively. In general, an event on a time section will map into a curve on the depth section which is common to all ellipses traced for the event from each shot geophone pair. For a variable velocity medium the underlying principles are the same except that the ellipses will be replaced by different curves.

### Conclusion

We presented three methods for common-shot migration. The methods are all designed for migration with vertical as well as lateral velocity variation. Clearly, extensive testing of the methods on different types of field data is required to evaluate their effectiveness. In actual applications one can foresee use of the fast ray tracing algorithm in a preliminary stage for an iterative determination of interval velocities, whereas the other two methods which operate on whole time sections can be used for actual data processing.

## Exact Kirchhoff Depth Migration of Unstacked Seismic Data S5.3

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A method for the two-dimensional depth migration of unstacked seismic data is presented. The method assumes that the subsurface velocity-depth structure is known and consists of 2-D isovelocity layers separated by interfaces which may be curved. The 2-D Rayleigh-Sommerfeld formulation of the Kirchhoff integral for nonmonochromatic scalar wave fields is used to migrate unstacked seismic data shot in the dip direction. Since the velocity-depth structure can have velocity variation laterally as well as with depth, ray tracing is used to evaluate the Kirchhoff integral exactly. The method requires that rays be traced from each depth sample to the surface, thereby defining migration curves on seismic field records. Amplitudes along the migration curves are corrected for the focusing and defocusing effects of geometrical divergence through ray tube theory. Also, a weighted aperture, centered on the specular reflection ray at the surface, is applied to the amplitudes along the migration curves to minimize noise. The energy summed along each migration curve is placed at its corresponding depth sample. Thus, by carrying out this procedure for all depth samples, a migrated depth section is determined. Since this method would be prohibitively time-consuming if each depth sample were treated individually, the FPS-100 array processor by Floating Point Systems, Inc. is used to carry out ray-tracing computations simultaneously for up to 1000 depth samples on a given depth section trace. It is hoped such an implementation of the Kirchhoff migration method will make its operation on unstacked data feasible.

### Introduction

Common-datum-point (CDP) stacking of seismic reflection data acquired through CDP profiling is generally carried out as part of the imaging process. However, when the subsurface geology is complex the CDP stacked events from steeply dipping reflectors are improperly imaged, resulting in resolution loss and attenuated amplitudes or even total loss of the reflection events. A possible solution to this problem is to drop CDP stacking as part of the imaging process and perform a full migration on the unstacked seismic data. Most full migration techniques utilize the Kirchhoff method because its implementation results in fast algorithms, a very