Parallel 15 ° finite difference migration with only two processing elements

t	A_0					t	A_0					t	A_0						
$\downarrow$	B_0	A_1				$\downarrow$	B_0	A_1				$\downarrow$	B_0	A_1					
	C_0	B_1	A_2				C_0	B_1	A_2				C_0	B_1	A_2				
	D_0	C_1	B_2				D_0	C_1	B_2	A_3			D_0	C_1	B_2	A_3			
	E_0	D_1	C_2				E_0	D_1	C_2	B_3	A_4		E_0	D_1	C_2	B_3	A_4		
	F_0	E_1	D_2				F_0	E_1	D_2	C_3	B_4		F_0	E_1	D_2	C_3	B_4	A_5	
				$\tau\rightarrow$							$\tau\rightarrow$								$\tau\rightarrow$

FIG. 5. Parallel 15° finite-difference migration when only a limited number of processing elements available. Here only two units used and migrated section generated in tiers.

nearly independent of step size. Therefore, we do not need to choose large extrapolation step sizes purely to save computational time and can freely employ the rather small extrapolation steps that are usually needed to realize the improved accuracy that high-dip algorithms offer.

A few small processors. Economic constraint usually limits the number of identical differencing stars one may cram into the parallel migration device. For example, on the SAXPY 1-M computer discussed by Levin and Parks (1985) there are 32 processing elements. In this event auxiliary storage will be needed to replace the lacking parallel components and to hold temporary results. A reasonable way to minimize (expensive) accesses to auxiliary storage is to mimic conventional 15-degree migration and do parallel sweeps as diagrammed in Figure 5. Alternatively, one may sweep horizontally instead of vertically (t-outer) or simply flip-flop processing elements along each subdiagonal, following the basic reverse time algorithm as much as possible.

Large processors (overlap). While it is not essential to provide each parallel unit with enough internal memory to hold the input and output vectors (and any temporary intermediate vectors that might arise), it can be desirable to do so. When the vectors are significantly shorter than the maximum length for which internal memory is designed, it is feasible to process multiple vectors inside the differencing unit. For example, one might identify each unit with a conventional pipelined array processor where it is advantageous to process multiple vectors to minimize I/O to the host computer or other processors. Here the processing element, as illustrated in Figure 6, can fill in a rectangular array computed from the sides or bottom before requiring new input. This allows us to sweep toward the diagonal in rectangular chunks, just as if they were simple differencing stars, only larger. Of course, edge effects have to be accounted for.

Further parallelism

We have now seen how migration decouples into independent constant τ vector calculations in constant t (i.e., $t' - \tau$) computational planes. Explicit algorithms, such as that of equation (6), increase parallelism even further by decoupling the individual x components of each vector. This further decreases the time needed for complete migration by a factor proportional to the number of traces, say another three orders of magnitude. In rough figures, if a migration of 1 000 by 1 000 point section normally takes half an hour on a conventional mainframe, then the reverse-time parallel organization potentially reduces this by about a factor of $1/2 \times 1\,000 = 500$ or to just under three seconds and an explicit migration gains an additional factor of 1 000, taking the time down to 3 ms. Even attaining only one percent of this theoretical gain brings migration into the fold of *interactive* tools available to the practicing geophysicist.

A	2	4
B	1	3
C	D	E

FIG. 6. Multiple input differencing star generated by repeated application of basic differencing star. This overlapped form useful for minimizing I/O when employing conventional large memory pipelined processors for parallel $x-t$ migration.

Conclusions

I have shown how to introduce a high degree of parallelism into conventional time-domain migration algorithms. This parallelism is even further enhanced for the class explicit migration algorithms and offers the attractive prospect of interactive migration to the geophysicist.

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Three-Dimensional Depth Migration by a Generalized Phase-Shift Method — S7.8

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We present a three-dimensional post stack migration algorithm which is based on an extrapolation in depth with the acoustic wave equation. The new method is designed to handle vertical as well as lateral velocity contrasts. The migration is carried out in the space-frequency domain. The wave equation is solved by a new expansion technique which virtually assures the same type of accuracy which is obtained with the phase-shift method for laterally uniform structures. The new method was implemented on the Cray XMP48 computer. The structure of the algorithm allows for easy utilization of vectorization and multitasking. Multitasking is achieved by allowing each task to perform calculations for different frequencies Vectorization is achieved by performing identical calculations along parallel lines within each frequency plane.

Basic equations

The depth migration is based on a variant of the temporally transformed acoustic wave equation given by

$$c^2\frac{\partial^2\tilde{P}}{\partial x^2} + c^2\frac{\partial^2\tilde{P}}{\partial y^2} + c\frac{\partial}{\partial z}\left(c\frac{\partial P}{\partial z}\right) = -\omega^2\tilde{P}. \quad (1)$$

x, y, and z denote Cartesian coordinates, $c(x,y,z)$ is the acoustic velocity, ω is the temporal frequency and $P(x,y,z,\omega)$ is the transform of the pressure field. Equation (1) departs from the classical wave equation in the vertical derivative term. This gives impedance matching for horizontal interfaces (Baysal et al., 1984). The generation of spurious downgoing energy at sharp interfaces is therefore reduced.

As in the 2-D case, it is convenient to recast (1) as a coupled first-order system given by

$$\frac{\partial}{\partial z}\begin{pmatrix}\tilde{P}\\ c\frac{\partial\tilde{P}}{\partial z}\end{pmatrix} = \begin{bmatrix}0 & \frac{1}{c}\\ -c\left(\frac{\omega^2}{c^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}\right) & 0\end{bmatrix}\begin{pmatrix}\tilde{P}\\ c\frac{\partial\tilde{P}}{\partial z}\end{pmatrix}. \quad (2)$$

After a spatial discretization in the horizontal coordinates and a specification of an approximation for the horizontal derivative operator, equation (2) transforms into an ordinary differential equation system given by

$$\frac{\partial}{\partial z}\left(\mathbf{U}\right) = \mathbf{A}\left(\mathbf{U}\right). \quad (3)$$

With N_x and N_y, respectively, denoting the number of seismic traces in the x and y-direction, $\mathbf{U}$ becomes a size of $2N_xN_y$ vector containing first the pressures and then the pressure derivatives. The size $2N_xN_y$ by $2N_xN_y$ matrix $\mathbf{A}$ is obtained from the operator on the rhs of (2).

The solution of (3) proceeds in depth increments. The increments should be chosen small enough to assume that the material properties do not vary in the vertical direction within them. As in the 2-D case the solution of (2) requires the generation of values of $c(\partial\tilde{P}/\partial z)$ on the surface before the depth extrapolation (Kosloff and Baysal, 1983). The final migration section is cumulated according to

$$P_{mig}(x,y,z) = \sum_{\omega}\tilde{P}(x,y,z,\omega), \quad (4)$$

(Kosloff and Baysal, 1983).

The generalized phase-shift method

The generalized phase-shift method departs from the formal solution of (3) given by

$$(\mathbf{U})_{z+dz} = e^{\mathbf{A}dZ}(\mathbf{U})_z. \quad (5)$$

Equation (5) is evaluated by the expansion

$$(\mathbf{U})_{z+dz} = e^{AdZ}(\mathbf{U})_z = \left[\sum_{k=0}^{M} C_kJ_k(R)Q_k\left(\frac{\mathbf{A}dZ}{R}\right)\right](\mathbf{U})_z, \quad (6)$$

where $C_0 = 1$, $C_k = 2$ for $k>0$, R is the range spanned by the eigenvalues of $\mathbf{A}dz$, and J_k are Bessel functions (Tal Ezer, 1984; Kosloff and Kessler, 1986). Q_k are matrix polynomials which are generated recursively by:

$$Q_{k+1}\mathbf{U} = Q_{k-1}\mathbf{U} + 2\left(\frac{\mathbf{A}dZ}{R}\right)Q_k\mathbf{U}, \quad (7)$$

with $Q_0 = \mathbf{U}$ and $Q_1 = \left(\frac{\mathbf{A}dz}{R}\right)\mathbf{U}$, (Kosloff and Kessler, 1986). For poststack migration, the velocity should be taken to equal half the actual velocity in the medium (Kosloff and Baysal, 1983). The solution of (6) is carried out for each frequency at all depths. The final section is calculated according to (4).

Implementation of the algorithm on the Cray XMP48 computer

The generalized phase-shift method allows for easy utilization of multitasking and vectorization. The fact that the downward continuation is carried out for each frequency separately enables each component of the four CPUs of the Cray XMP48 computer to operate on a different frequency. Within a single CPU most of the computational effort involves calculation of horizontal derivatives by FFTs which is a highly vectorizable operation.

Results of applying the new migration algorithm indicate that it is highly accurate and efficient compared to schemes based on numerical integration techniques for ordinary differential equations, such as the Runga-Kutta and Predictor-Corrector methods. In fact for laterally uniform structures the new algorithm gives the same results as the ordinary phase shift method (Gazdag, 1978) to within the precision of the computer.

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Seismic 8— S/N Improvement and Land Acquisition

Low Seismic Frequencies: Acquisition and Utilization of Broad-Band Signals Containing 2-8 Hz Reflection Energy S8.1

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Seismic resolution is improved by increasing signal bandwidth toward both higher and lower frequencies. The low frequencies are particularly important in inversion of the seismic trace; their deliberate acquisition can significantly improve the accuracy of surface-derived velocity sections. This is demonstrated with real