Modeling and Studying Wavefields Scattered by Fractal Interfaces

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Nowadays a fractal geometry, introduced by B.Mandelbrot plays a central role in studies of complicated natural object. Faults, folds, salt tectonic structures, porous rocks (including that in reservoir) have a fractal natural. The paper is devoted to computation studies of properties of wavefield formed by interface having fractal geometry. Analysis of time slices and seismograms for impulse with different apparent frequencies and for multifolded system of acquisition allows to came to the following conclusions.

A regular system of waves is formed by a fractal interference. The system looks as if it were formed in multilayered medium. In case of a fractal the most part of observed waves are diffracted ones. A number of waves increases as apparent frequency of a wave impulses also increases. Effective centers of diffraction are slightly moving in a zone surrounding the fractal, depending on location of an acquisition system. A wave reflected by same effective surface, closed to smoothed fractal, is formed by rather large angles of incidence.

The diffraction centers also radiate waves in a lower part of a medium. A formation of wave refracted by smoothed fractal starts by rather smaller angles of incidence. Trajectories of propagation of reflected and refracted waves can be calculated with a help of ray formulas. There is possibility to identify a interface with fractal geometry using an atlases of its homeomorphic images, constructed with a help of homeomorphic stacks of different types.

A new approach, based on a concept of a so called fractal geometry and introduced by B.Mandelbrot (1982) attract a great attention of researchers working in different areas of nature sciences, technique, medicine, art etc. Fractals also applied in geology, geography and geophysics. Nowadays landscapes, mountains, islands, rivers, sediments, rock grain and many other are studied from a fractal geometry point of view [Feder 1988]. Seismic objects, like faults, folds, porous rocks (including that in reservoirs) salt tectonic structures have a fractal nature(Hewett 1986, Leary 1992).

One should expect that a process of wave propagation in a medium with a fractal geometry is very complicated and have certain unusual features. Many theoretical and experimental papers are devoted to studies of the process of propagation of optical, electro-magnetic, sound and seismic waves in media of this type. For example, Okuvo and Aki 1983, Wie and Aki (1985), found evidence supporting assumption that in the lithosphere may be fractal surfaces. In the ground of all these studies known for us, lies a stochastical approach, mainly based on fundamental results obtained by Berry, 1979. However it is very impotent (especially for seismic studies) to tray to consider a process of formation of wavefields in medium with fractals in the deterministic formulation. Unfortunately, it looks that an deterministic analytic technique meets serious obstacles connected with a nonexistence of the first derivatives for a fractal surface.

In the HI approach it was assumed (Gelchinsky 1989) that fractal surfaces could be approximated by a thin zone formed by scattered effective diffractors locations of which depend on a disposition of sources and receivers, recording the wavefield. Hence it follows that locally wavefield, formed by a fractal and recorded by a fixed regular system, should have a complicated but regular character.

This investigation was performed with aim to check this hypothesis and to study of properties of wavefield from a deterministic standpoint. A numerical method, and computer software developed by Koslov et all , was chosen for calculation. The Weierstrass-Mandelbrot (W-M) function was used for modeling a fractal interface(Barry and Lewis 1980). This function W(x) is presented in a form

$$W(x,D)=\mathrm{Re}\sum_{n=-\infty}^{n=\infty}\frac{(1-e^{ijnx})e^{i\phi_n}}{\tau^{(2-D)n}} \qquad (1)$$

($1<D<2$, $\tau>0$) where D is a so called Housdooff-Besicovitch (fractal) dimension of the graph W(x) (D=1 corresponds to a 1D curve) τ is a parameter and the arbitrary phases ϕ_n can be chosen in deterministic or stochastic way. The fixed values of ϕ was use in this calculation. The frequencies τ^n spanning the range zero to infinity in a geometric progression, that is the sence, in which the W-M function possesses no scale . The series for W(x) converges, but the series for dW/dx does not.

Detailed analytical and computational study of the W-M function is presented in the paper by Berry and Hewis 1980. Here only some graphs of a fractal interfaces, corresponding to different values of D and trend, are shown on the Fig.1. The relationship used for their calculation, was slightly different from the formula (1) because in seismic application, it is necessary to compare curves with different D, corresponding to the same trend. However the trend $W_0(x,D)$ of (1) defined by the formula

$$W_0(x,D) \approx \frac{x^{2-D}\,\Gamma(D-1)\cos(\pi(2-D)/2]}{(2-D)\ln(\tau)} \qquad (2)$$

depends on a value of D . There for, the "pure" fractal part W(x,D) determined by the formula

$$W(x,D) = W(x,D) - W_0(x,D) \qquad (3)$$

was added to the trend $W_0(x,D=1.95)$.

The effect of varying D is clear. As values of D increase, a behavior of W(x,D), as a function of coordinate x becomes more complicated. The curve, corresponding to D=1.95 looks like a some thin stripe intricated by large picks. An impression arises that values of the picks could be choosen as a measure of dimension. It is not quite correct, because it is possible to diminish values of this picks varying the interval for argument x, without changing values of dimension D.

The model, consisting the two interfaces: the first fractal with D=1.95 and the second plane boundary, was chosen for calculation (Fig.3). The Berlage impulse with dominant frequency 50Hz was used by calculation, spacing is equal to 3.8m. The time slices of a calculated wavefield and the seismograms are shown on the Fig.3 and 4. At first sight it is clear, that a regular systems of waves is formed by the fractal interface. These waves can be easy traced on the time slices and seismograms. An examination of the slices allows to came to following conclusions. At the first moment (Fig.3b,c), when a direct wave reaches the fractal, a wave of diffracted (or may be scatted) type is formed and begin to propagate back to the free surface without any slights of strong interference with the following diffracted waves, formed as other parts of a front of the direct wave, touched the fractal. Any sight of a reflected waves is not seen in areas surrounding the first diffracted waves, which are corresponds to small angle of incidence close to a" normal reflection".(Fig. 3b). Only at the next moments (approximation from t≈0.4 sec.,Fig.3c) a formation of a reflected wave starts. The reflected waves becomes clear and intensifies at the following moments, while angle of incidence of direct waves, increases (Fig.3d and 3e). It can be interpreted as the diffracted waves, corresponding to large angles of incidence, interfere strong each with other and forms reflected wave. However, large number of separated diffracted or scattered waves still exists (Fig.3e,4b).

A wavefield picture, observed in a low part under the fractal interface is partly similar to describe above situation, partly different from it. A large number of separated diffracted waves is observed in the lower halfspace. A part of them, corresponding to small angles of incidence, propagated to the second interface and back separately. However the refracted wave is formed by very small angles of incident(Fig.3b).

Despite a complicated picture, observed in the lower parts of the time slices and corresponding to an interference between the waves propagating down and up, the waves, reflected from the second layer can clear be detected in these parts of the slices (Fig.3e,f). The refracted-reflected wave can also be traced on the upper part of the time slices (Fig.3f) and the seismograms(Fig.4b). A conclusion following from the observation of the formation of wave refracted by a fractal interface is very important. It means that a trajectory of a wave transmitted through a fractal interface can be calculated according to the ray formulas, taking in account surface obtained by smoothing a fractal.

Similar wavefield pictures were observed on other time slices and seismograms calculated for different dispositions of sources and receivers. A part of calculation were made for wave impulses having the same shape but different dominant frequencies. The typical seismograms are shown on the Fig.4. It is easy to see that a regular system of waves are observed in a wide frequency range (from 25Hz till 150Hz), a number of observed waves increases with a impulse dominant frequency. Although travel time curves of this waves chance not very essentially, dynamic characteristics of the waves undergo much more stronger alteration. A certain preliminary estimation shows that locations of effective centers of diffraction (scattering) are not permanent, depending on apparent frequency and disposition of an acquisition system.

Wavefields formed by interfaces of different types of a fractal geometric were also calculated. The conclusions following from consideration of all calculated wafefields are. A regular system of waves are formed by a interface having a fractal geometry. Wavefield pictures observed in multilayered medium and formed by a fractal interface, bear a resemblance in many respects. However in the case of a fractal it is possible to identify observed waves as diffracted or scattered ones. A number of waves increases as a apparent frequency of a wave impulse also increases. Effective centers of diffraction are slightly moving in a zone surrounding the fractal, depending in location of an acquisition system and an apparent frequency of wave impulse. A wave reflected by same effective surface, close to the smoothed fractal, is formed by rather large angles of incidence.

The diffraction centers radiated waves in all directions, including the other side of a fractal a formation a wave refracted by a fractal starts by rather smaller angles of incidence. The reflected and refracted waves are stronger then the diffracted waves. Trajectories, along which these waves propagate , can be calculated with a help of ray method.

All said means that the hypothesis, used by construction of atlases of homeomorphic images (Fig.5b, Gelchinsky 1989) is correct. There is possibility to distinguish interfaces, having a fractal geometry, from smooth or corrugated surfaces using these atlases, constructed with a help of homeomorphic stack of different types (Gelchinsky 1989, in this volume).

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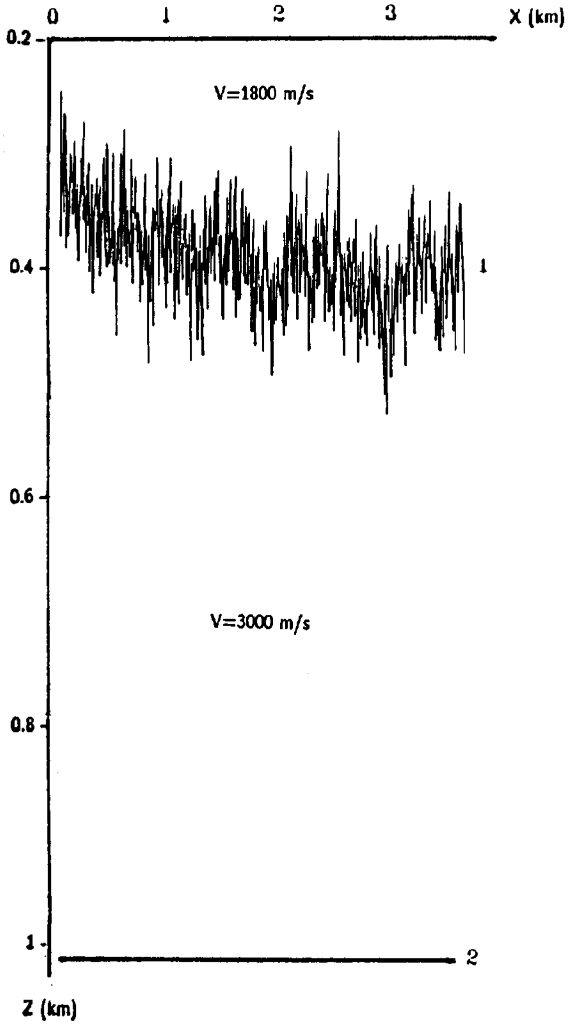

Fig.1 Model for the calculations synthetic seismograms.
1) Fractal boundary
2) Horizontal boundary

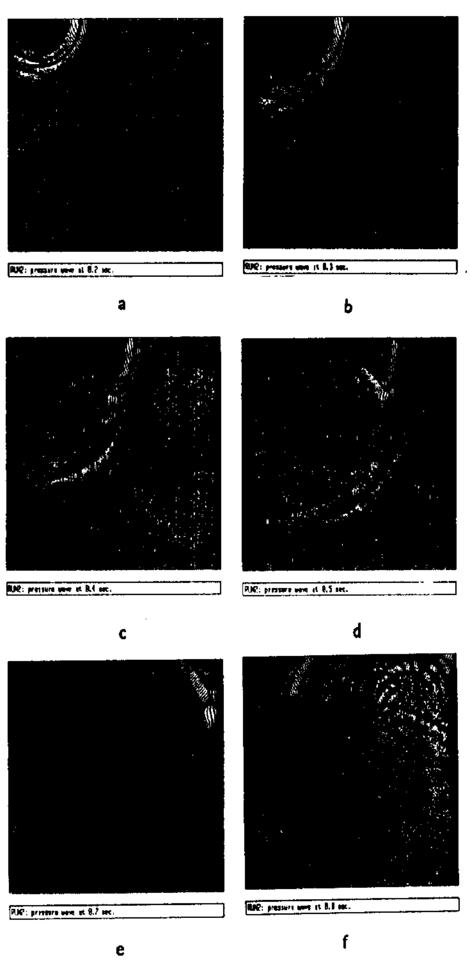

Fig.2 Description of the wavefield in the media at the different time

a-0.2s.; b-0.3s.; c-0.4s.;

d-0.5s.; e-0.7s.; f-0.8s.

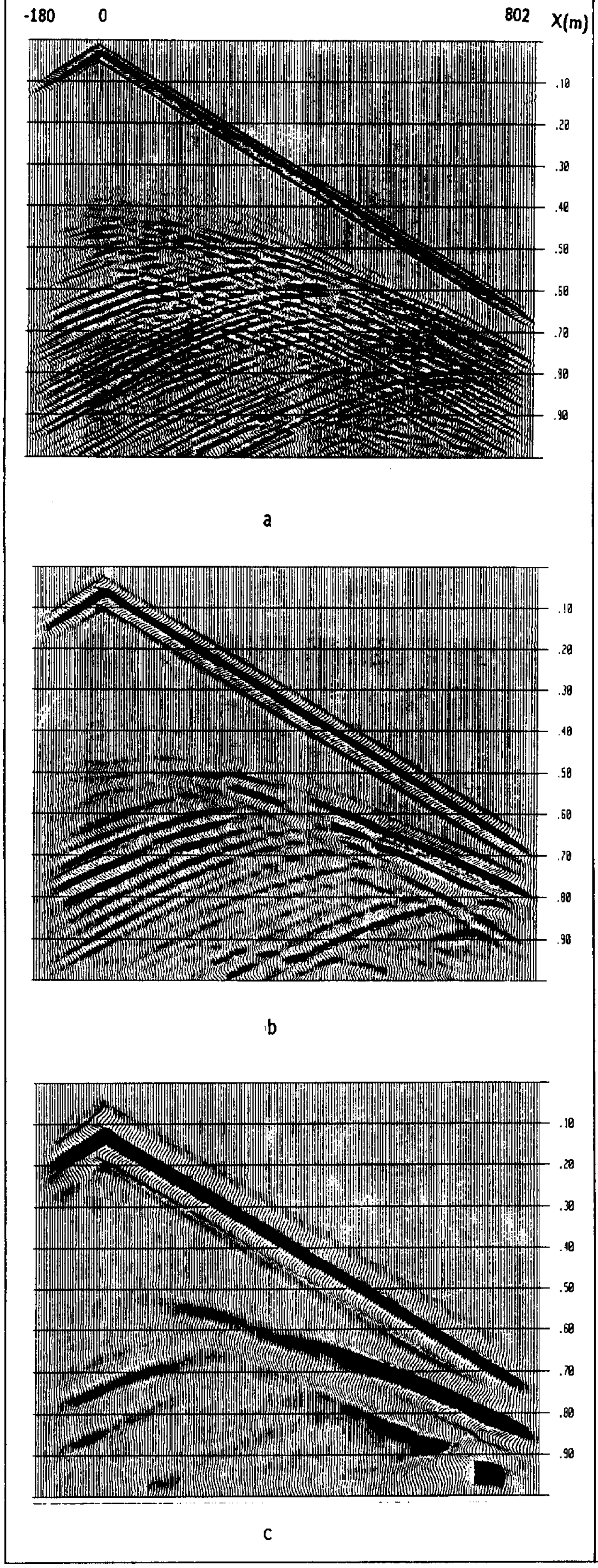

Fig.3 Seismograms of reflected and diffracted waves corresponding to the model on the Fig.1
The dominant frequency of the source:
a) 100 Hz; b) 50 Hz; c) 25Hz