Mixed-grid solution of the 3-D Eikonal equation

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SUMMARY

Calculation of 3D traveltimes is needed for the application of Kirchhoff integral based prestack depth migration. *3D* prestack imaging requires a significantly large amount of computation, where a major part is dedicated to traveltime calculations. Therefore, a practical method of calculating traveltimes is needed.

Our algorithm was developed with the following objectives: *(a)* the numerical scheme should be simple and efficient, *(b)* the traveltimes should represent arrival times of body waves, and (c) the calculated traveltime field should be accurate for steep dip structures.

To achieve these objectives, we (a) directly solve the *3D* Eikonal equation on a Cartesian coordinate system, *(b)* modify the over-critical arrivals on a special cylindrical coordinate system, and (c) apply a spatial convolutional operator to overcome errors introduced by low order approximations.

INTRODUCTION

Several authors have presented solutions to the Eikonal equation in three dimensional space. Schneider (1993) and Fowler (1994) solved the Eikonal equation using a spherical coordinate system, and Jervis et al. (1994) modified the method for the calculation of minimum traveltimes. However, the works which inspired our development were the *2D* solutions presented by Reshef and Kosloff (1986), and by Van Trier and Symes (1988).

In the algorithm presented in this paper, propagation of the traveltime solution from depth z to depth $z+dz$ is achieved by three steps. First, downward extrapolation is applied. Second, the locations of the over-critical waves are identified and the traveltimes associated with these locations are replaced by body waves arrivals. Third, a balancing operator is applied to the solution, fixing inconsistent traveltimes resulting by numerical approximations.

The resulting traveltime cubes consist of body wave arrival times only and are accurate for structures that include steep dips and severe velocity variations. Moreover, since the algorithm requires that only a small amount of information need to be stored, the solution scheme becomes very efficient when applied on massively parallel computers.

SOLUTION SCHEME

Time field propagation starts by downward extrapolating the time field $T(x,y,z)$ from depth z to depth $z+dz$. The Eikonal equation

$$\frac{\partial T}{\partial z} = \sqrt{\frac{1}{C^2} - \left(\frac{\partial T}{\partial x}\right)^2 - \left(\frac{\partial T}{\partial y}\right)^2}$$

is directly solved on a Cartesian coordinate system $G(x,y,z)$:

$$G(x, y, z) = G(i \cdot dx, j \cdot dy, k \cdot dz)$$

where dx, *dy*, and *dz* represent spatial sampling in the x, y, and z directions, respectively; $1 < i < Nx$, $1 < j < Ny$, $1 < k < Nz$, and Nx, Ny, and Nz represent the number of grid points in x, y, and z directions respectively. A finite difference operator is used for application of spatial derivatives, and a Runge-Kutta algorithm is used for numerical integration (Reshef, 199 1).

The resulting traveltime field, $T(x,y,z+dz)$, contains arrival times of body-waves as well as of head-waves. In order to replace the traveltimes of the (first to arrive) head-waves with the appropriate body-waves (direct arrivals), we transform $T(x,y,z+dz)$ to a special cylindrical coordinate system $H(r, \theta, z)$:

$$H(r, \theta, z) = H[dr(i,j), d\theta(i,j), k \cdot dz]$$

which follows the direction of propagation of the wave-fronts (see figure 1). The cylindrical coordinate system $H(r,\theta,z)$, is constructed using a local radius dr and a local angle $d\theta$. $d\theta$ is the angle of propagation of the wave fronts on a grid node of G; *dr* is the distance from a grid node of G to the intersection with the neighbor grid cell; and *dz* is the same as in G.

Locating all the x, y nodes of G where head-waves arrivals exist, we exclude the traveltime information represented by these nodes. Using the upwind condition (Van Trier and Symes, 1991). we use the transformed traveltime field $T(r,\theta,z+dz)$ and stretch the solution along each local radius, back to the above x, y node on G. The modified traveltime function $T'(r,\theta,z+dz)$ now contains arrival times associated with body-waves only.

The traveltime field T' might contain some noise, resulting from numerical approximations. In order to assure. stability, differencing functions might need some degree of smoothness. Therefore, as a

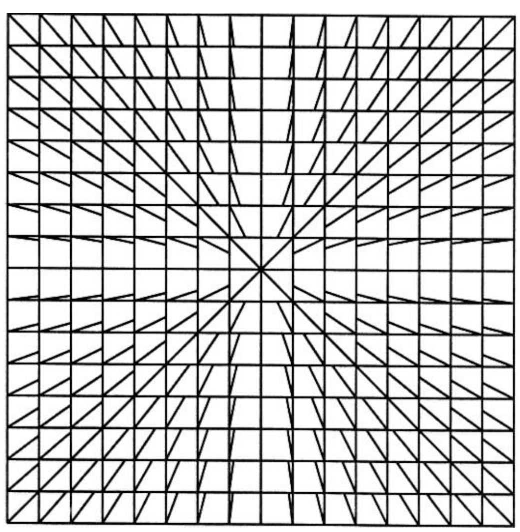


Figure 1: Mixed grid. Horizontal s/ice through the 3D mesh for a case of circular wave-fronts. In the solution scheme, downward extrapolation is done on the cartesian nodes, and head-wave to body-wave corrections are made on the cylindrical nodes.

final step, we apply a *2D* spatial convolutional operation, aiming to correct local inconstancies:

$$T''(x,y,z+dz) = T'(x,y,z+dz) * Q(x,y)$$

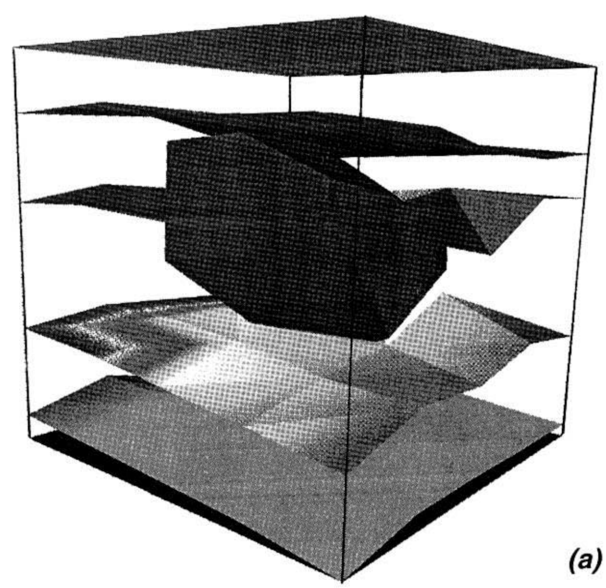
where $Q(x,y)$ is the 3x3 matrix:

$$Q(x,y) = \begin{bmatrix} 0.0 & 0.125 & 0.0 \\ 0.125 & 0.5 & 0.125 \\ 0.0 & 0.125 & 0.0 \end{bmatrix}$$

This smoothing convolutional operator permits us to smooth the traveltimes instead of the difference operators. Derived from a Hanning window, Q is very smooth in the wave-number domain, has high tangency at zero and Nyquist wave-numbers, and therefore does not bias the calculated traveltimes.

EXAMPLE

Figure 2 shows a model, containing a salt body embedded in a series of semi-flat layers. The velocity in the layers ranges from 2000 m/s up to 3500 m/s, and the salt velocity is 4000 m/s.
Figure 3 shows the full traveltime cube calculated for a source positioned at the surface. in the center of the model. The cube size is 200x200x300.
As we can see, the traveltime contours are circular and symmetric on the surface. At the subsurface,



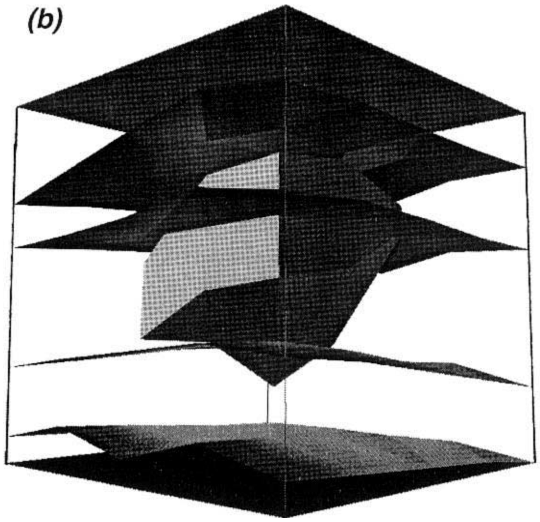


Figure 2: Salt model. (a) View from the side. (b) View from the bottom.

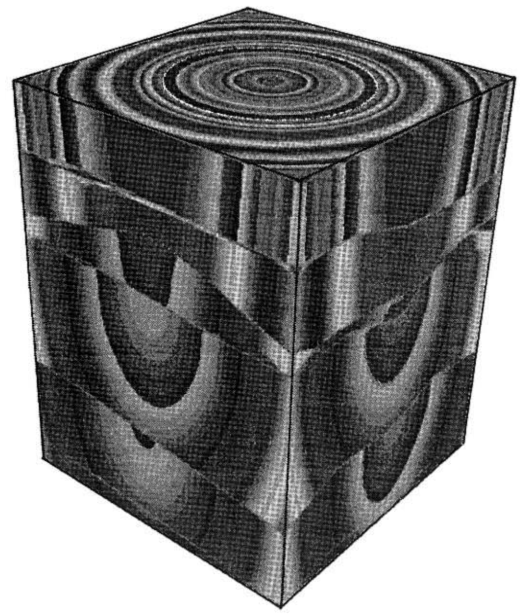


Figure 3: Traveltime cube calculated for the salt mode/presented in figure 2. The cube size is 200x200x300.

the contours are continuous within the layers, but are not continuous when crossing layer boundaries. Figure 4 is a vertical cut through the full size cube. In this view we can see the behavior of body waves when propagating from low to high velocity layers. Figure 5 show two horizontal slices of the traveltime cube. In these figures we can observe how the wave fronts follow the velocity structure. The wave-fronts are not circular any more, and discontinuity occurs at velocity interfaces.

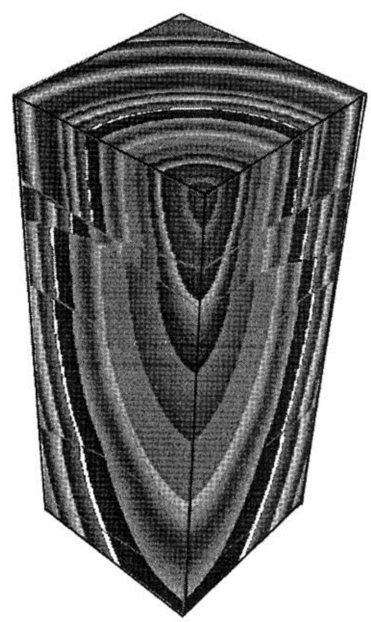


Figure 4: Vertical slice of the cube shown in figure 3. The traveltime field consists of body waves only.

CONCLUSIONS

A new and efficient method for calculation of *3D* traveltime functions for prestack depth migration has been presented. The algorithm is based on solving the *3D* Eikonal equation on a mixed grid. The traveltime function is extrapolated using a cartesian grid, and head-wave arrivals are replaced by body-waves using a special cylindrical grid. The resulted traveltime cube contains arrival times associated with body waves only.

Since the solution method is very efficient, traveltime cubes can be calculated very fast, enabling the use of the algorithm scheme in a *3D* prestack depth migration program.

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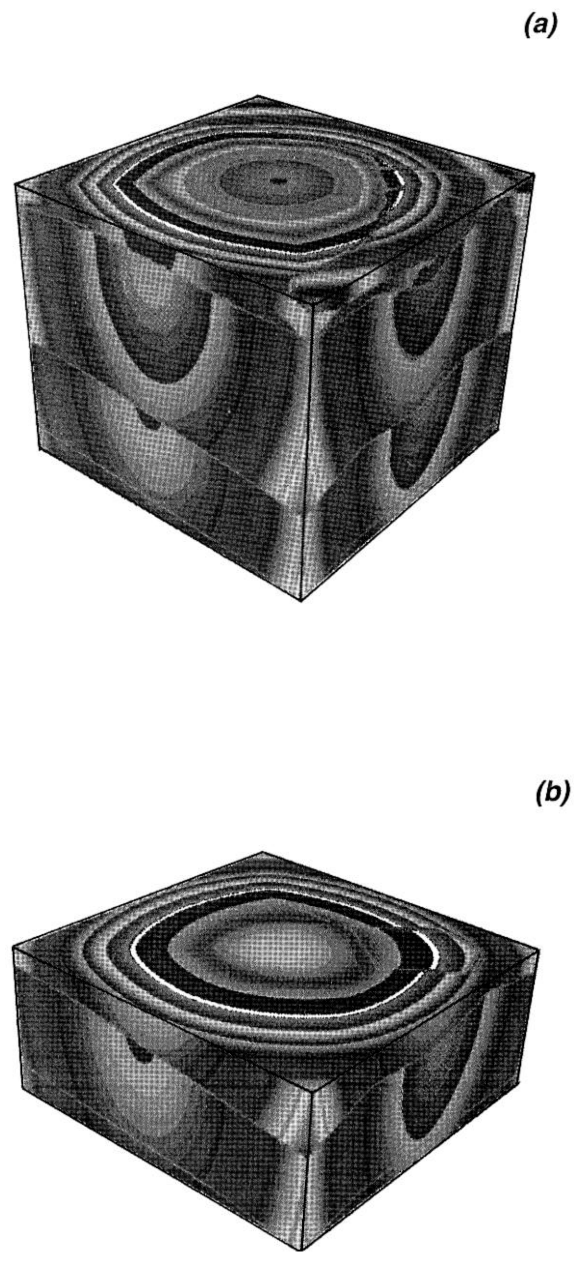

Figure 5: Horizontal slices of the cube shown in figure 3. (a) Slice at about 1000 m. (b) S/ice at about 2000 m. Traveltime wave-fronts follow *the structure of the velocity field.*