Three dimensional wave equation depth migration by a direct solution method

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Summary

We propose a new method for wave equation depth migration which is carried out by depth stepping in the space-temporal frequency domain. The propagation of the solution is based on a rational expansion of the formal solution to the acoustic wave equation. The expansion coefficients are calculated by a filter design approach. The method is not based on one way wave equations or on perturbations to constant velocity solutions to the wave equation, but rather it uses the constant density variable velocity wave equation as the basis. The method has a good steep dip response and can handle strong lateral variations in the subsurface velocity.
The new method is tested against the Sigsbee data set and the 3D SEG salt model.

Introduction

Most downward-continuation methods for surface recorded data are based on some approximation to the variable velocity acoustic wave equation. These approximations include use of a one way wave equation, instead of the acoustic wave equation, as in finite difference migration, use of perturbations to constant velocity solutions as in the phase screen methods, or application of spatially variant filters which are designed from constant velocity solutions, as with explicit operator methods. When these approximations are used in wave equation migration, they may limit the ability to handle steep dips or strong lateral velocity gradients.

This article presents a new space-frequency domain downward continuation method termed the direct solution method. This method is quite different from the above mentioned approaches in that it uses the exact acoustic wave equation for the actual subsurface velocity. In the approach, the propagation of the solution from one depth level to the next is carried out by a rational expansion of the one way evolution operator. As with explicit methods, the expansion coefficients are calculated by a filter design approach. The solution is generated by applying the wave equation operator to the data a number of times for obtaining both the positive power terms of the expansion as well as the rational terms. The new method has a very good steep dip response and handles lateral velocity variability in a natural manner.
In the following sections we first present the theory behind the direct solution method. The new method is then tested against a number of synthetic examples which have become standard in exploration seismology, namely, the Sigsbee data set and the SEG salt model. The results of the tests show that the direct solution method is capable of successfully imaging structures with complicated velocity variation.

The Formal Solution

The direct solution method is based on a depth extrapolation of the temporarily transformed constant density acoustic wave equation which writes,

$$\frac{\partial^2 \tilde{p}}{\partial z^2} = -\frac{\omega^2}{c^2}\tilde{p} - \frac{\partial^2 \tilde{p}}{\partial x^2} - \frac{\partial^2 \tilde{p}}{\partial y^2}. \quad (1)$$

$\tilde{p}(x, y, z, \omega)$ denotes the Fourier transform of the pressure field $p(x, y, z, t)$, ω is the temporal frequency, x and y are the horizontal coordinates, z is the depth, and $c(x, y, z)$ is the velocity.
The equation may be spatially discretized on a uniform mesh (x, y, z) followed by a selection of a second derivative approximation to transform the equation into a system of ordinary differential equations as follows,

$$\frac{d^2\mathbf{p}(z)}{dz^2} = -\frac{\omega^2}{c_0^2}\mathbf{D}\mathbf{p}(z), \quad (2)$$

where,

$$\mathbf{D} = \frac{c_0^2}{c^2(x, y, z)}\mathbf{I} + \frac{c_0^2}{\omega^2}\nabla^2. \quad (3)$$

c_0 is a constant velocity (usually chosen as the minimum velocity in each layer), and $\mathbf{I}$ is the identity matrix.
We assume that within each layer (z , $z+dz$) the velocity is vertically constant. Then the upward propagating solution to (2) within a layer can formally be written as,

$$\mathbf{p}(z+dz) = e^{i\frac{\omega dz}{c_0}\sqrt{\mathbf{D}}}\mathbf{p}(z). \quad (4)$$

Rational expansion of the solution

The operator $\mathbf{D}$ in (3) contains both positive and negative eigenvalues which respectively correspond to propagating and evanescent waves. There are very small eigenvalues in the transition region between evanescent and non evanescent waves. Because of these components, a polynomial expansion of the exponential operator in (4) would converge very slowly. Consequently we chose a rational expansion of the solution given by,

$$\mathbf{p}(z+dz)=\sum_{i=0}^{m_p} a_i \mathbf{D}^i \mathbf{p}(z)+\sum_{i=0}^{m_n-1} b_i \frac{1}{\mathbf{D}-\beta_i \mathbf{I}} \mathbf{p}(z).$$

(5)

a_i, b_i and β_i are pre calculated coefficients. m_p+1 is the number of positive power terms in the expansion, and m_n is the number of rational terms. The next section explains the procedure for calculation of the coefficients.
Equation (5) is used for propagating the solution from one depth level to the next. Given the pressure field $\mathbf{p}(z)$, $\mathbf{DP}$, $\mathbf{D}^2\mathbf{p}$, ... $\mathbf{D}^{\mathbf{m_p}}\mathbf{p}$ are calculated by recursively applying the operator $\mathbf{D}$. The first sum in (5) can then be cumulated.
Each rational term in the second sum is calculated by solving a linear equation of the form,

$$(\mathbf{D}-\beta_i \mathbf{I})\mathbf{v}=\mathbf{p}(z). \qquad (6)$$

This calculation can be quite costly and therefore we have used only one rational term in the evaluation of (5). In the implementation, the Laplacian is calculated by the Fourier method and equation (6) is solved by the iterative pre-conditioned flexible GMRES method. The generation of one rational term in (6) required approximately twenty applications of the operator $\mathbf{D}$. Typically five positive power terms and one rational term are required for propagating the solution from one depth level to the next.

Calculation of the expansion coefficients

When c_0 in (3) is chosen as the minimum velocity in the layer, the eigenvalues of $\mathbf{D}$ approximately range between one and (sometimes large in magnitude) negative values. This range can be reduced somewhat by spatial Fourier filtering of the wave number components for which $k^2 > f\omega^2/c_0^2$, where k is the wave number, and $f>1$ is a safety factor. The filtered components correspond to evanescent waves. As the filter cutoff wave number is conservative and is based on the minimum velocity in the layer, some evanescent components can remain in the solution. However, the expansion coefficients are designed to prevent these components from exponentially growing out of bounds.

For a given values of $\alpha=\omega\, dz/c_0$, the first step of the design is to find the coefficients of the rational approximation,

$$e^{i\alpha\sqrt{x}} \approx \frac{c_0+c_1x+c_2x^2....+c_{m_p+m_n}x^{m_p+m_n}}{1+d_1x+d_2x^2+....d_{m_n}x^{m_n}}. \qquad (7)$$

For a given x_j, equation (7) can be recast as a linear equation for the coefficients c_i and d_i given by,

$$c_0+c_1x_j+c_2x_j^2....+c_{m_p+m_n}x_j^{m_p+m_n}-d_1x_je^{i\alpha\sqrt{x_j}}- -d_2x_j^2e^{i\alpha\sqrt{x_j}}+......-d_{m_n}x_j^{m_n}e^{i\alpha\sqrt{x_j}}=e^{i\alpha\sqrt{x_j}}. \qquad (8)$$

A least squares fit of (8) for values of x_j in the range $\varepsilon < x_j \le 1$ is used for calculating the coefficients c_i and d_i, where ε is a small positive number. For negative x we need to assure stability by requiring that the norm of the approximation be less than one. This was achieved by adding three negative x_j points in the least squares fit.

After determination of c_i and d_i, the values of β_i in (5) can be found by calculating the roots of the denominator of (7). In order to assure reasonable convergence of the GMRES scheme for solving (6), small β_i values are scaled upwards by multiplication by a positive number greater than one. After the determination of β_i, the coefficients a_i and b_i in (5) are determined by a second least square fit of the equation,

$$e^{i\alpha\sqrt{x}} \approx a_0+a_1x+a_2x^2+...+a_{m_p}x^{m_p}+ +b_1\cdot\frac{1}{x-\beta_1}+....+b_{m_n}\cdot\frac{1}{x-\beta_{m_n}}. \qquad (9)$$

Example: The SMAART Sigsbee2a data set

The first test of the new migration algorithm was the 2D Sigsbee data set without a free surface. The velocity section is shown in Fig 1. The data set consisted of 500 shot files with a shot spacing of 150 feet. Each shot contained 348 channels with an offset range between zero and 26025 feet. The recording time was 12 sec at a sample rate of 8 msec. The CMP spacing was 37.5 feet. The imaging was carried out by 2D common shot migration.
Fig 2 shows a migrated depth section which was obtained with the Direct Solution method. The figure shows that the Direct Solution algorithm images the salt boundary and the sub salt sediments quite well.

Example: The SEG salt model

The Direct Solution Method is tested against the 3D SEG salt model data set. The survey consisted of 389 inlines (191-589), and 355 crosslines (80-434). The inline spacing and the crossline spacing was 20m.
The data set contained 4874 shot files. The number of receivers per shot was 134. The shot spacing in the inline direction was 80m, and the shot spacing in the crossline direction was 160m. The receiver spacing in the inline and crossline directions was 40m and the number of streamers was 8.
Fig 3 shows the velocity section for inline 360. The velocity volume used in the migration was obtained after a small degree of smoothing was applied to the original velocity volume (in retrospect this smoothing was not necessary).
Fig 4 shows a section of the migrated image along inline 360. This figure demonstrates that considering the difficulties with this data set, the Direct Solution method was able to obtain a good image of this model.

Conclusions

We have presented a new migration method which appears to have theoretical advantages of current migration techniques in that it uses the acoustic wave equation directly for the real subsurface velocity. Our experience indicates that this method yields slightly better results than the Phase Shift plus Correction method with multiple background velocities. However in terms of cpu time the Direct Solution Method is about three times slower than the latter. It still remains to be seen in which situations this extra effort is justified.

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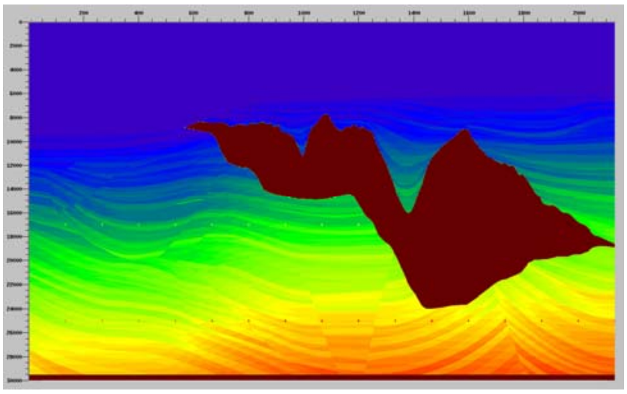

Figure 1: Sigsbee2a velocity section.

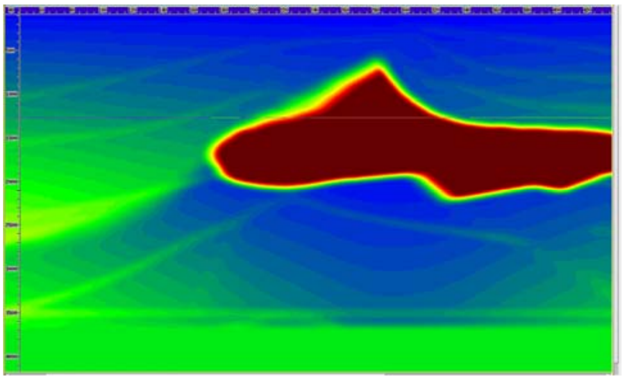

Figure 3: Velocity section at inline 360 of the SEG salt model.

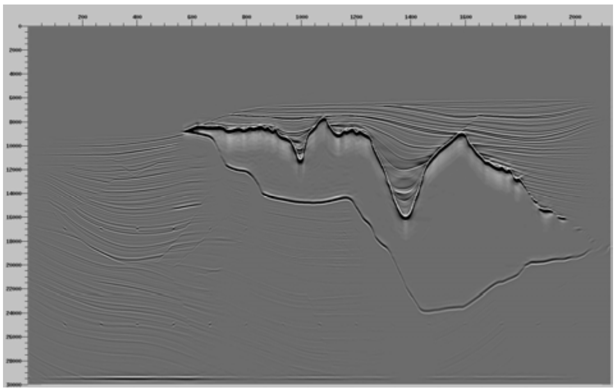

Figure 2: Sigsbee2a migrated image.

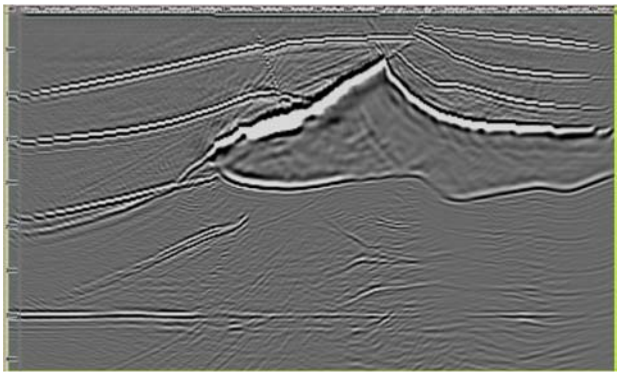

Figure 4: Migrated image at line 360 of the SEG salt model.