

Discussion

On: "Migration by Fourier transform," by R.H. Stolt (January 1978 GEOPHYSICS, 43, p. 23-48).

In this paper (page 34) the migrated section is given by

$$\phi(X, D, D) = (1/2\pi) \int dp \int dk B(p, k) e^{-i(pX - kD)} \dots \quad (51)$$

where,

$$B(p, k) = \{1/\sqrt{(1 + p^2/k^2)}\} \cdot A\{p, (kc/2)\sqrt{1 + p^2/k^2}\} \dots \quad (52)$$

However, I find a factor $(c/2)$ is missing in either equation (51) or (52). The necessary derivation follows.

The migrated section is given by equation (50) of this paper,

$$\phi(X, D, D) = (1/2\pi) \int dp \int d\omega A(p, \omega) e^{-i(pX - \sqrt{4\omega^2/c^2 - p^2} \cdot D)}$$

$$\text{We have, } k = \sqrt{4\omega^2/c^2 - p^2} \quad (i)$$

$$\text{then, } k^2 = 4\omega^2/c^2 - p^2 \quad (ii)$$

$$\text{or, } 2kdk = (4/c^2) \cdot 2\omega d\omega \quad (iii)$$

$$\text{Now, from (ii), } 4\omega^2 = (p^2 + k^2)c^2$$

$$\text{or, } \omega = (kc/2)\sqrt{1 + p^2/k^2} \quad (iv)$$

$$\text{Therefore, } d\omega = (kc/2)(1/\sqrt{p^2 + k^2}) dk$$

$$\text{or, } d\omega = (c/2) \cdot dk/\sqrt{1 + p^2/k^2} \quad (v)$$

Now by change of variable from ω to k

$$\phi(X, D, D) = (1/2\pi) \int dp \int (c/2)/\sqrt{(1 + p^2/k^2)} dk \cdot$$

$$A\{p, (kc/2)\sqrt{1 + p^2/k^2}\} e^{-i(pX - kD)} \quad (vi)$$

If we consider,

$$B(p, k) = \{1/\sqrt{(1 + p^2/k^2)}\} \cdot A\{p, (kc/2)\sqrt{1 + p^2/k^2}\}, \quad (52)$$

equation (vi) becomes

$$\phi(X, D, D) = \{(c/2)/2\pi\} \int dp \int dk \cdot B(p, k) e^{-i(pX - kD)} \quad (vii)$$

Thus, the migrated section $\phi(X, D, D)$ in equation (vii) has an additional factor of $c/2$.

$$\text{Introducing } B1(p, k) = (c/2)B(p, k), \quad (viii)$$

$$\phi(X, D, D) = (1/2\pi) \int dp \int dk \cdot B1(p, k) \cdot e^{-i(pX - kD)}. \quad (ix)$$

Consequently, although the shifted frequency ω' remains unchanged, the factor for change of scale is given by,

$$\begin{aligned} B1(p, k)/A(p, \omega) &= (c/2) \cdot 1/\sqrt{1 + p^2/k^2} \\ &= (c/2) \cdot k/\sqrt{(p^2 + k^2)} \\ &= (c/2)\omega'/\omega. \end{aligned}$$

The equation (ix) for the migrated section is equivalent to equation (51) given in this paper, but the change of scale will be $(c/2) \cdot \omega'/\omega$ instead of ω'/ω as given in this paper.

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Reply by the author to Somaditya Dutta

Mr. Dutta is correct. To be consistent with equation (50), equation (51) should have a factor of $c/2$ out front.

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